Now, differentiate $\boldsymbol{S}(u,v)$ directly, first w.r.t. u,

$$\boldsymbol{S}_u(u,v) = \frac{\partial}{\partial u}\boldsymbol{S}(u,v) =$$

$$= \sum_{j=0}^{m} N_{j,q}(v)\left(\frac{\partial}{\partial u}\sum_{i=0}^{n} N_{i,p}(u)\boldsymbol{P}_{ij}\right)$$

$$= \sum_{j=0}^{m} N_{j,q}(v)\left(\frac{\partial}{\partial u}\boldsymbol{C}_j(u)\right)$$

where,

$$\mathbf{C}_j(u) = \sum_{i=0}^{n} N_{i,p}(u)\,\mathbf{P}_{ij}$$

Applying the derivative control point formula to each $\mathbf{C}_j(u)$ yields,

$$\mathbf{S}_u(u,v) =$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{m} N_{i,p-1}(u)\,N_{j,q}(v)\,\mathbf{P}^{(1,0)}_{ij}$$

where,

$$\boldsymbol{P}_{ij}^{(1,0)} = p\left(\frac{\boldsymbol{P}_{i+1,j} - \boldsymbol{P}_{ij}}{u_{i+p+1} - u_{i+1}}\right)$$

$$U^{(1)} =$$

$$= \{\underbrace{0, \ldots, 0}_{p}, u_{p+1}, \ldots, u_{r-p-1}, \underbrace{1, \ldots, 1}_{p}\}$$

$$V^{(0)} = V$$

Analogously,

$$\boldsymbol{S}_v(u, v) =$$

$$= \sum_{i=0}^{n} \sum_{j=0}^{m-1} N_{i,p}(u) N_{j,q-1}(v) \boldsymbol{P}_{ij}^{(0,1)}$$

where,

$$\boldsymbol{P}_{ij}^{(0,1)} = q\left(\frac{\boldsymbol{P}_{i,j+1} - \boldsymbol{P}_{ij}}{v_{j+q+1} - v_{j+1}}\right)$$

$$U^{(0)} = U$$

$$V^{(1)} =$$

$$= \{\underbrace{0, \ldots, 0}_{q}, v_{q+1}, \ldots, v_{s-q-1}, \underbrace{1, \ldots, 1}_{q}\}$$

Applying first the *u*- and then the *v*-derivative yields,

$$\boldsymbol{S}_{uv}(u, v) =$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} N_{i,p-1}(u) N_{j,q-1}(v) \boldsymbol{P}^{(1,1)}_{ij}$$

where,

$$\boldsymbol{P}^{(1,1)}_{ij} = q\left(\frac{\boldsymbol{P}^{(1,0)}_{i,j+1} - \boldsymbol{P}^{(1,0)}_{ij}}{v_{j+q+1} - v_{j+1}}\right)$$

$$U^{(1)} =$$

$$= \{\underbrace{0, \ldots, 0}_{p}, u_{p+1}, \ldots, u_{r-p-1}, \underbrace{1, \ldots, 1}_{p}\}$$

$$V^{(1)} =$$

$$= \{\underbrace{0, \ldots, 0}_{q}, v_{q+1}, \ldots, v_{s-q-1}, \underbrace{1, \ldots, 1}_{q}\}$$